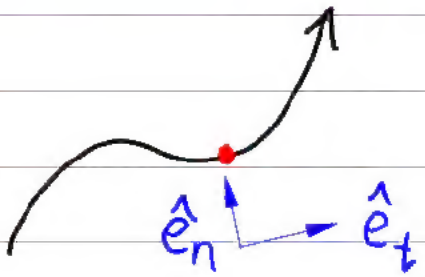
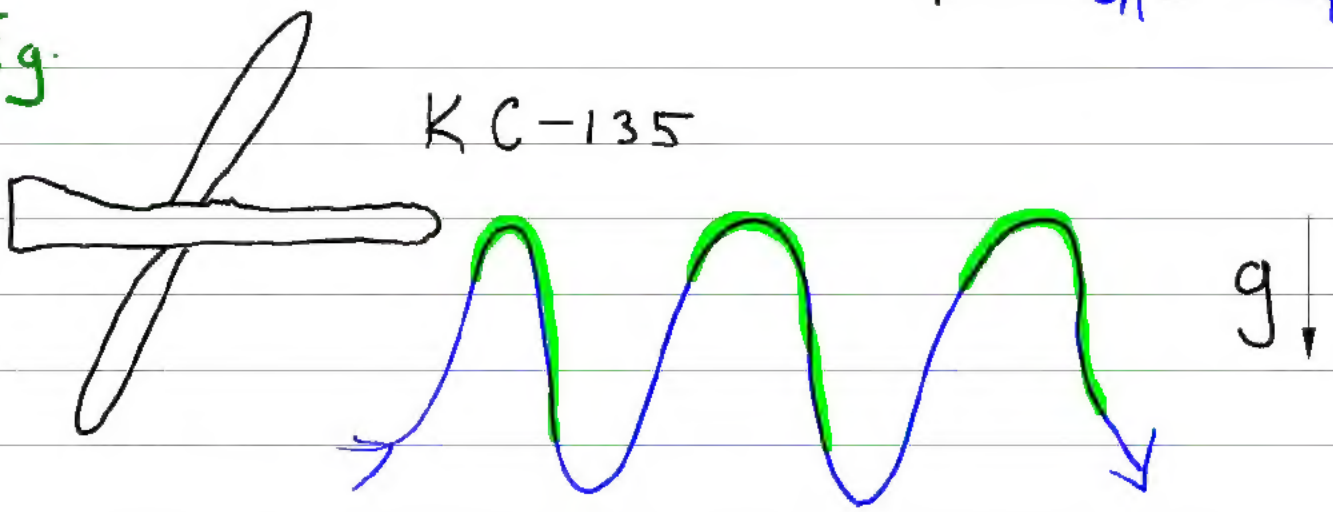
Meriam 2/5: n-t coordinates (continued)

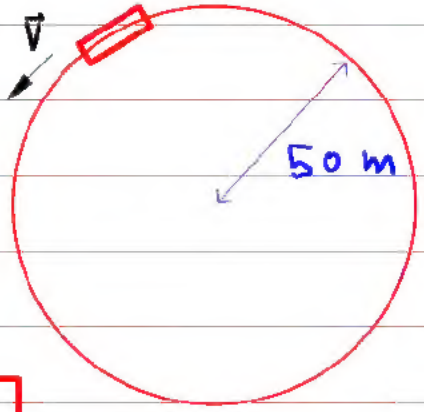
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$



Eg.



Eg. A car on a circular track has a speed of 20 m/s, and is slowing down at a rate of $0.7\ m/s^2$. Find its acceleration after 8 seconds.



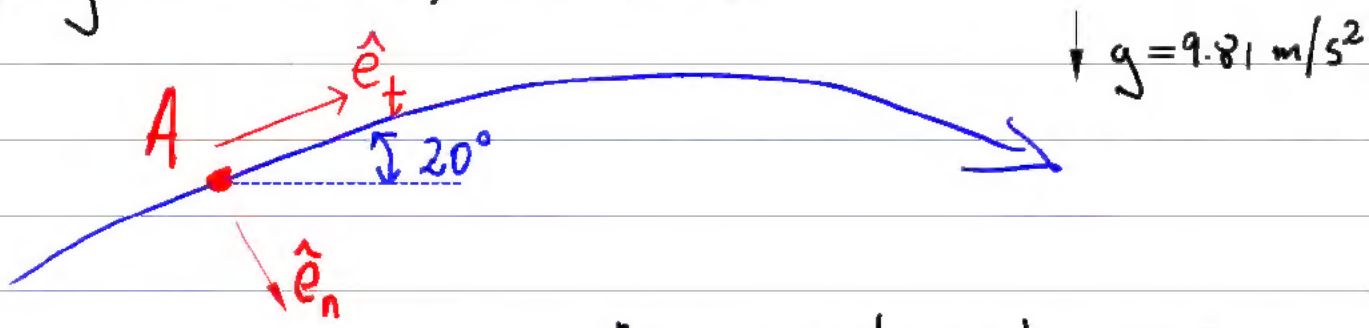
$$\vec{a} = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

$$= -0.7\hat{e}_t + \frac{(20 - 8\times 0.7)^2}{50\ m}\hat{e}_n$$

$$\vec{a} = -0.7\,\hat{e}_t + 4.15\,\hat{e}_n \quad m/s^2$$

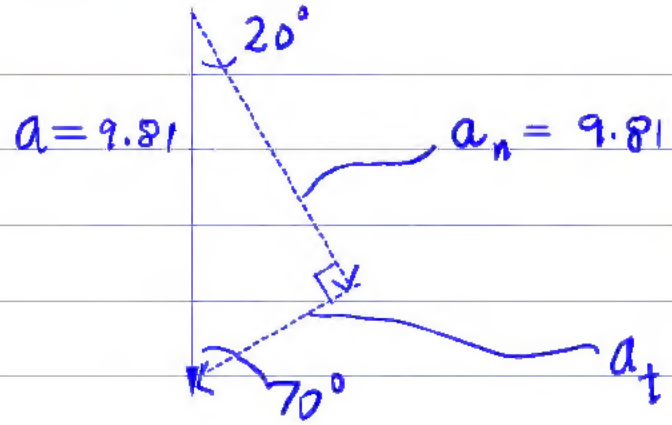
+ve or −ve always +ve

E.g. Find radius of curvature of rock projectile at "A", if $V = 50\ m/s$



Assume wind resistance = 0

Answer: $a_n = v^2/\rho \rightarrow \rho = V^2/a_n$



"projection" of a vector

$$\rightarrow \boxed{\rho = v^2/a_n = 50^2/9.22 = 271\ m}$$

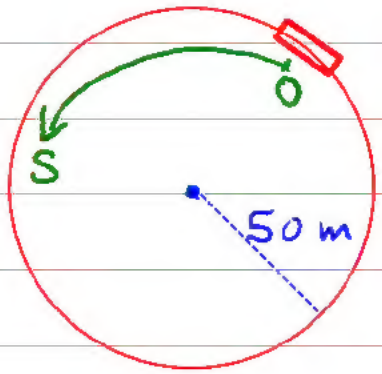
$$\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\,\hat{e}_n$$

$$= a_t\,\hat{e}_t + a_n\,\hat{e}_n$$

$$= -9.81\cos(70°)\,\hat{e}_t + 9.81\cos(20°)\,\hat{e}_n$$

$$\boxed{\vec{a} = -3.36\,\hat{e}_t + 9.22\,\hat{e}_n\ m/s^2} = -9.81\,\hat{j}\ m/s^2$$

E.g. Truck on circular track, $R = 50$ m.
It starts at position $s = 0$, with $v_0 = 4$ m/s
If $\dot{v} = 0.05\,s$ m/s^2, find v and $\vec{a}$ at $s = 10$ m



- Do NOT use $v^2 = v_0^2 + 2a_0\Delta s$

- acceleration is a function of position s

$$\frac{dv}{dt} = a_t \rightarrow dt = \frac{dv}{a_t}$$

$$\frac{ds}{dt} = v \rightarrow dt = \frac{ds}{v}$$

$$\Big\} \; a_t\,ds = v\,dv$$

$$\int_{s=0}^{s=10} a_t\,ds = \int_{v_0=4}^{v_{final}} v\,dv$$

$$\int_0^{10} (0.05)(s)\,ds = \int_4^{v_{final}} v\,dv$$

$$(0.05)\frac{s^2}{2}\Big|_{s=0}^{s=10} = \frac{v^2}{2}\Big|_4^{v_{final}}$$

$$\frac{(0.05)}{2}(100-0) = \frac{(v_{final}^2 - 16)}{2}$$

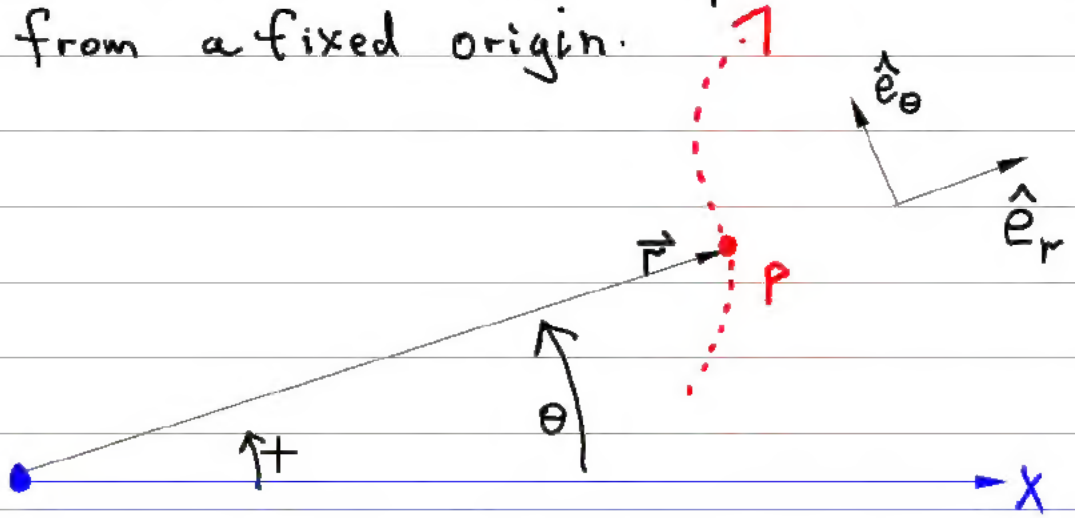
$$\rightarrow \boxed{v_{final} = \sqrt{21} = 4.58 \text{ m/s}}$$

$a_n = \frac{v^2}{\rho} = \frac{21}{50} = 0.42$; $a_t = \dot{v} = (0.05s) = (0.05)(10) = 0.5$

$$\boxed{\vec{a} = 0.5\,\hat{e}_t + 0.42\,\hat{e}_n \text{ m/s}^2} \quad \text{in x-y coordinates?}$$

Meriam 2/6: Polar (r-θ) Coordinates

The polar coordinate system is good for tracking the motion of particles in terms of distance and direction from a fixed origin.



Define;

- $\vec{r}$ = position of P relative to origin
- r = $|\vec{r}|$ = distance from origin to P (always + ve)
- $\hat{e}_r$ = unit vector, $\parallel$ to $\vec{r}$, pointing away from origin
- $\hat{e}_\theta$ = unit vector, $\perp$ to $\hat{e}_r$, pointing in the positive rotational direction.
- θ = angular position between reference axis and $\vec{r}$ (can be +ve or -ve)

- By convention, we often choose the x-axis as the reference axis for defining θ.

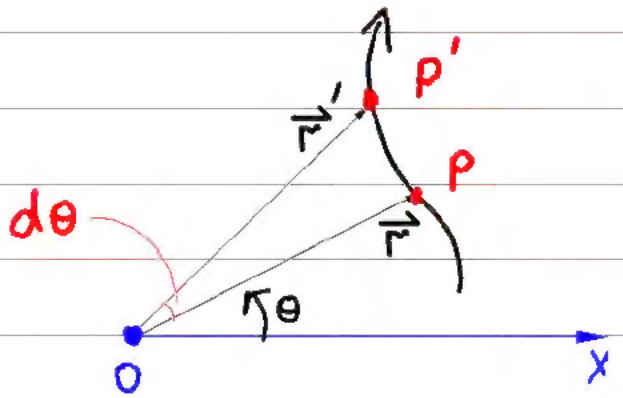
- By convention, we usually pick the counter-clockwise direction as +ve ↺

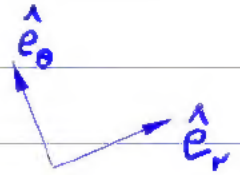
Find expressions for $\vec{r}$, $\vec{v}$, and $\vec{a}$:

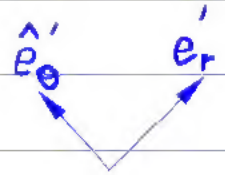
position: $\vec{r} = r\,\hat{e}_r$

velocity: $\vec{v} = \dot{\vec{r}} = \frac{d}{dt}(r\,\hat{e}_r)$

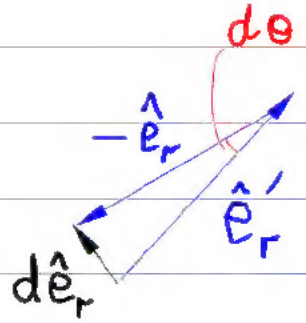
$$\vec{v} = \dot{r}\,\hat{e}_r + r\,\frac{d\hat{e}_r}{dt}$$







$d\hat{e}_r = \hat{e}_r' - \hat{e}_r$



$d\theta$ (in radians) $= \frac{|d\hat{e}_r|}{1} \rightarrow d\hat{e}_r = d\theta\,\hat{e}_\theta$

$$\frac{d\hat{e}_r}{dt} = \frac{d\theta\,\hat{e}_\theta}{dt} = \dot{\theta}\,\hat{e}_\theta$$

Substitute that result into ★

$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

or $\vec{v} = v_r\,\hat{e}_r + v_\theta\,\hat{e}_\theta$ where $v_r = \dot{r}$; $v_\theta = r\dot{\theta}$

Note: (i) v_r & v_θ can be +/−

(ii) Express $\dot{\theta}$ in radians/second (s^{-1})

Eg. A carousel turns at 8 rpm. A woman initially 4m from the center walks toward the center at 2 m/s. Find $\vec{v}$

Answer: $\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$

where $r = 4$ m

$\dot{r} = -2$ m/s

$$\dot{\theta} = \frac{8 \text{ rev}}{\text{min}} \cdot \frac{2\pi \text{ radians}}{\text{rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 0.84 \ s^{-1}$$

$$\vec{v} = -2\,\hat{e}_r + 3.35\,\hat{e}_\theta \text{ m/s}$$

Are v_r & v_θ constant ?

Summary

- x-y coordinates :

$$\vec{r} = x\hat{i} + y\hat{j}$$
$$\vec{v} = \dot{x}\hat{i} + \dot{y}\hat{j}$$
$$\vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$$

- n-t coordinates

$$\vec{v} = v\,\hat{e}_t$$
$$\vec{a} = \dot{v}\,\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

- polar (r-θ) coordinates

$$\vec{r} = r\,\hat{e}_r$$
$$\vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$
$$\vec{a} = ?$$

Next Lecture

- Polar coordinates: $\vec{a}$ (Meriam 2/6)
- Numerical Examples/Solved Problems